Q#1 Kinematics

The velocity of a particle is given as

$$\boxed{\mathbf{v}(t) = 10t^2\hat{\mathbf{x}} + 5t^3\hat{\mathbf{y}}} \qquad \text{(Eq. 1-1)}$$

where $\boxed{\hat{\mathbf{x}}, \hat{\mathbf{y}}}$ are the direction vectors.

A) What is the speed at $t = 2$?

B) What is the acceleration at $t = 2$?

C) What is the average velocity in the x-direction between $t = 0$ and $t = 2$?

A#1 Kinematics

The velocity of a particle is given as

$$\boxed{\mathbf{v}(t) = 10t^2\hat{\mathbf{x}} + 5t^3\hat{\mathbf{y}}} \qquad \text{(Eq. 1-1)}$$

where $\boxed{\hat{\mathbf{x}}, \hat{\mathbf{y}}}$ are the direction vectors.

A) What is the speed at $t = 2$?

The speed is the magnitude of the velocity vector.

$$\boxed{\mathbf{v}(t=2) = 40\hat{\mathbf{x}} + 40\hat{\mathbf{y}}} \qquad \text{(Eq. 1-2)}$$

$$\boxed{\begin{aligned} |\mathbf{v}(t=2)| &= \sqrt{40^2 + 40^2} \\ &= 56.568542494924 \end{aligned}} \qquad \text{(Eq. 1-3)}$$

B) What is the acceleration at $t = 2$?

$$\boxed{\mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt} = 20t\hat{\mathbf{x}} + 15t^2\hat{\mathbf{y}}} \qquad \text{(Eq. 1-4)}$$

$$\boxed{\mathbf{a}(t=2) = 40\hat{\mathbf{x}} + 60\hat{\mathbf{y}}} \qquad \text{(Eq. 1-5)}$$

C) What is the average velocity in the x-direction between $t = 0$ and $t = 2$?

$$\boxed{\bar{\mathbf{v}} = \frac{\Delta\mathbf{x}}{\Delta t} = \frac{\mathbf{x}_f - \mathbf{x}_i}{t_f - t_i}} \qquad \text{(Eq. 1-6)}$$

$$\boxed{\begin{gathered}\mathbf{v}(t) = \frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} \quad \Rightarrow \quad \mathbf{x}(t) = \int_0^t \mathbf{v}(t')\,\mathrm{d}t' \\ \mathbf{x}(t) = \frac{10}{3}t^3\hat{\mathbf{x}} + \frac{5}{4}t^4\hat{\mathbf{y}}\end{gathered}} \qquad \text{(Eq. 1-7)}$$

$$\boxed{\begin{aligned}&\mathbf{x}(t=0) = 0\hat{\mathbf{x}} + 0\hat{\mathbf{y}} \\ &\mathbf{x}(t=2) = \frac{80}{3}\hat{\mathbf{x}} + 20\hat{\mathbf{y}} \\ &\bar{\mathbf{v}} = \frac{\frac{80}{3}\hat{\mathbf{x}} + 20\hat{\mathbf{y}} - 0}{2-0} = \frac{40}{3}\hat{\mathbf{x}} + 10\hat{\mathbf{y}} - 0\end{aligned}} \qquad \text{(Eq. 1-8)}$$

$$\boxed{\bar{\mathbf{v}}_x = \frac{40}{3} = 13.33\overline{3}} \qquad \text{(Eq. 1-9)}$$

Q#2 Fluid Flow

If water is flowing through a tube that necks down in size determine (A) the velocity and (B) the pressure of the smaller section if the area of the bigger section is 1 cm^2, the area of the smaller section is 0.5 cm^2, the pressure in the bigger tube is 3 atm, and the velocity of the water flow in the big tube is 10 m/sec.

A#2 Fluid Flow

If water is flowing through a tube that necks down in size determine (A) the velocity and (B) the pressure of the smaller section if the area of the bigger section is 1 cm^2, the area of the smaller section is 0.5 cm^2, the pressure in the bigger tube is 3 atm, and the velocity of the water flow in the big tube is 10 m/sec.

$$\boxed{\begin{array}{ll} P_1 = 3\text{ atm} = 3\cdot 1.013\cdot 10^5\text{ PA} & P_2 = ? \\ A_1 = 1\text{ cm}^3 & A_2 = 0.5\text{ cm}^3 \\ v_1 = 10\,\dfrac{\text{m}}{\text{s}} & v_2 = ? \end{array}} \qquad \text{(Eq. 2-1)}$$

A) The amount of liquid flowing into or out of the tube per unit time is a function of the fluid density ρ, the velocity, and the cross sectional area of the pipe. Water is usually considered incompressible, thus it has a constant density:

$$\boxed{\rho_{\text{water}} = 1\frac{\text{gm}}{\text{cm}^3} = 10^3\,\frac{\text{kg}}{\text{m}^3}} \qquad \text{(Eq. 2-2)}$$

The amount of fluid entering the tube per unit time has to be equal to the amount leaving the tube, since the fluid is not being used up or created in the middle. In fact the amount of fluid flowing past any point must be equal to that flowing past any other point for the same reason.

$$\boxed{\begin{array}{l} \rho A_1 v_1 = \rho A_2 v_2 \\ A_1 v_1 = A_2 v_2 \end{array}} \qquad \text{(Eq. 2-3)}$$

$$\boxed{v_2 = \frac{A_1 v_1}{A_2}} \qquad \text{(Eq. 2-4)}$$

$$\boxed{v_2 = 20\,\frac{\text{m}}{\text{s}}} \qquad \text{(Eq. 2-5)}$$

B) Bernoulli's equation related pressure, velocity and height for a fluid, in a manner similar to the kinetic and gravitational energy of a particle are related:

$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2} \quad \text{(Eq. 2-6)}$$

In our case, the height is the same for point 1 and point 2, so those terms cancel out or can be set to zero height, leaving us with:

$$\boxed{\begin{aligned} &P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \\ &P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) \end{aligned}} \quad \text{(Eq. 2-7)}$$

$$\boxed{\begin{aligned} P_2 &= 3\cdot 1.013\cdot 10^5 \text{ PA} \quad + \frac{1}{2}\left(10^3 \frac{\text{kg}}{\text{m}^3}\right)(100-400)\frac{\text{m}^2}{\text{s}^2} \\ &= 3\cdot 1.013\cdot 10^5 \text{ PA} \quad -1.5\cdot 10^5 \text{ PA} \\ &= 1.539\cdot 10^5 \text{ PA} \\ &= 1.5192 \text{ atm} \end{aligned}} \quad \text{(Eq. 2-8)}$$

Q#3 Ramp, Pulley, Two Blocks

Two objects of masses m_1 and m_2, are connected by a cable that is wrapped around a pulley as in Figure 3-1.

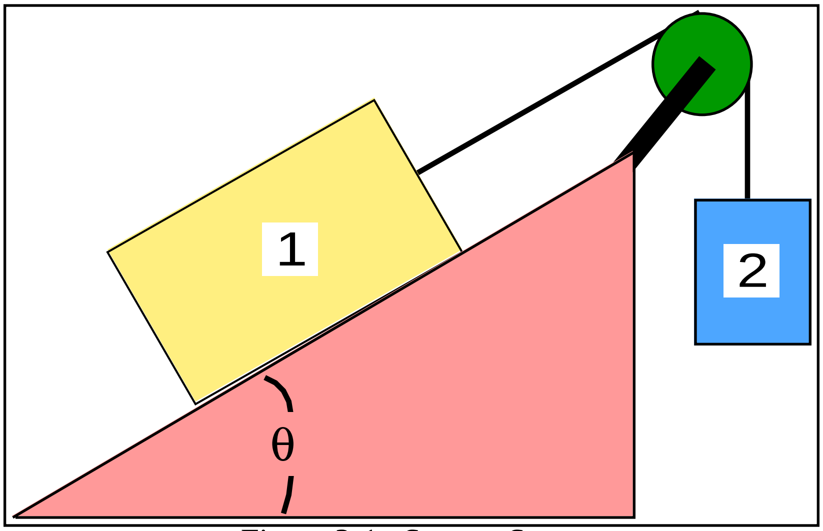

Figure 3-1. System Setup

A) Develop an expression which relates the two masses when the system is in equilibrium for arbitrary angle of inclination of the plane.

B) If the coefficient of friction between m_1 and the plane is μ then what is the maximum mass m_{2max} which can be attached to the pulley for the system to stay in equilibrium (not move)?

A#3 Ramp, Pulley, Two Blocks

Two objects of masses m_1 and m_2, are connected by a cable that is wrapped around a pulley as in Figure 3-1.

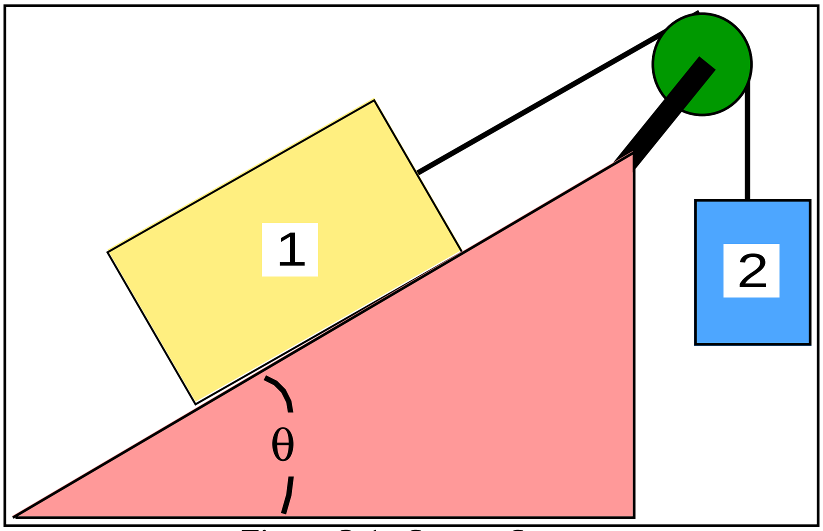

Figure 3-1. System Setup

A) Develop an expression which relates the two masses when the system is in equilibrium for arbitrary angle of inclination of the plane.

We must assume that the pulley is frictionless and massless and that the cable is massless in order to proceed. Rather than just doing the equilibrium case (no acceleration) we will look at the more general case of a non-zero acceleration.

If there is no friction for the sliding block, the only forces are the blocks weight due to gravity, the reaction force from the surface of the ramp (the "normal" force perpendicular to this surface) and the tension in the cable pulling up the ramp. We can draw the free body diagram in Figure 3-2. Note how we have decided on directions for our coordinate system as well as the direction of the resultant acceleration, however these are not drawn on the FBD.

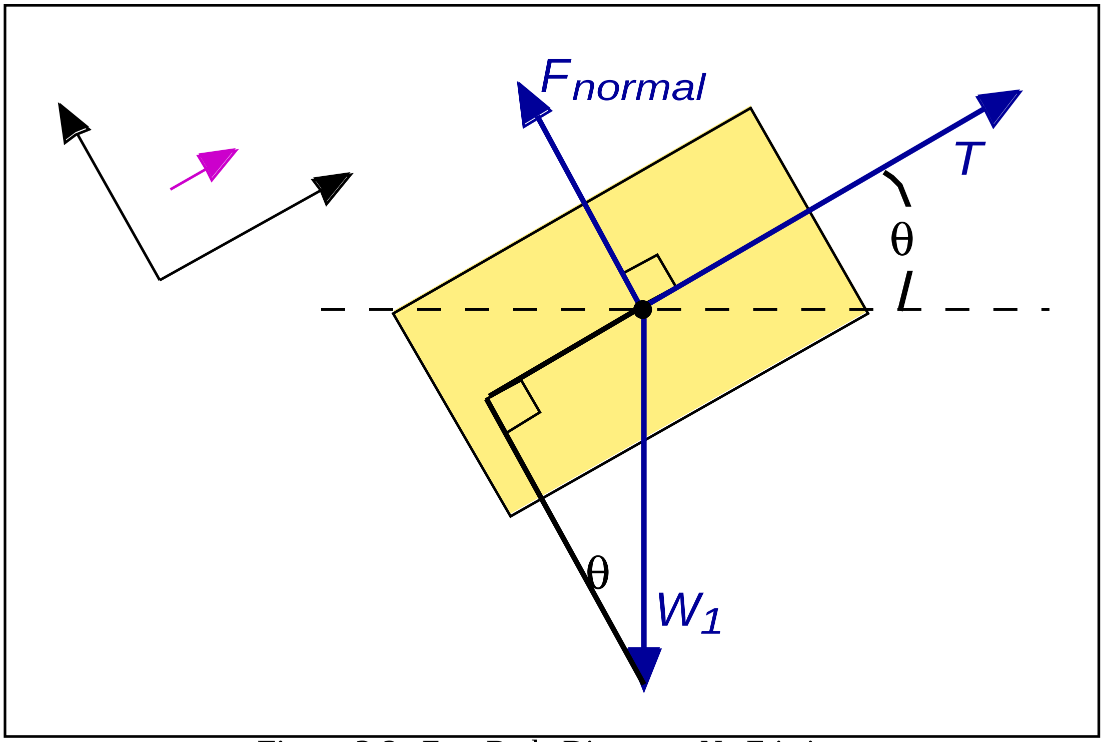

Figure 3-2. Free Body Diagram - No Friction

From the FBD we proceed with Newton's second law:

$$\boxed{\begin{array}{l} \sum_i \mathbf{F}_i = m\mathbf{a} \\ \sum_i F_{x\,i} = ma_x \\ \sum_i F_{y\,i} = ma_y \end{array}} \qquad \text{(Eq. 3-1)}$$

We know that the acceleration, *a*, is in the *x*-direction, with no component in the *y*-direction, so we get:

$$\boxed{\sum_i F_{x\,i} \quad \Rightarrow \quad T - W_1 \sin\theta = m_1 a} \qquad \text{(Eq. 3-2)}$$

$$\boxed{\sum_i F_{x\,i} \quad \Rightarrow \quad F_{\text{normal}} - W_1 \cos\theta = 0} \qquad \text{(Eq. 3-3)}$$

For the hanging block we can make a FBD with only two forces, the block's weight and the tension in the cable.

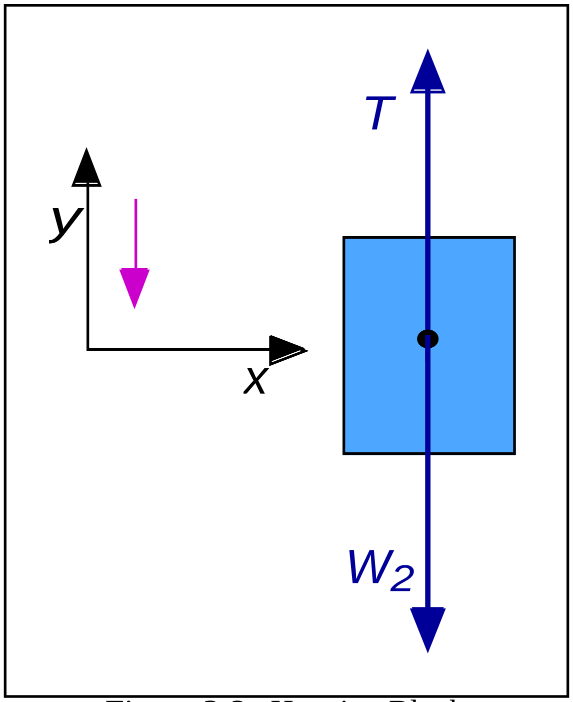

Figure 3-3. Hanging Block

$$\boxed{\begin{aligned} \sum_i \mathbf{F}_i &= m\mathbf{a} \\ \sum_i F_{xi} &= ma_x \\ \sum_i F_{yi} &= ma_y \end{aligned}} \tag{Eq. 3-4}$$

We know that the acceleration, a, is in the negative y-direction, with no component in the x-direction, so we get:

$$\boxed{\sum_i F_{xi} \quad \Rightarrow \quad 0 = 0} \tag{Eq. 3-5}$$

$$\boxed{\sum_i F_{xi} \quad \Rightarrow \quad T - W_2 = -m_2 a} \tag{Eq. 3-6}$$

We also know what the two weights are:

$$\boxed{\begin{aligned} W_1 &= m_1 g \\ W_2 &= m_2 g \end{aligned}} \qquad \text{(Eq. 3-7)}$$

Putting Equation 3-7 into Equation 3-6 and Equation 3-2, and combining them to eliminate *T*, we get:

$$\boxed{\begin{array}{l} \;\;T - m_1 g \sin\theta = m_1 a \\ -T + m_2 g \qquad\;\; = m_2 a \\ \hline m_2 g - m_1 g \sin\theta = m_1 a + m_2 a \end{array}} \qquad \text{(Eq. 3-8)}$$

$$\boxed{a = g\frac{m_2 - m_1 \sin\theta}{m_1 + m_2}} \qquad \text{(Eq. 3-9)}$$

For the equilibrium case, as the question asked, the acceleration is zero which will only occur if:

$$\boxed{m_2 = m_1 \sin\theta} \qquad \text{(Eq. 3-10)}$$

B) If the coefficient of friction between m_1 and the plane is μ then what is the maximum mass m_{2max} which can be attached to the pulley for the system to stay in equilibrium (not move)?

If we have friction between the ramp and the sliding block, there is an extra force to add to the FBD. The direction of this frictional force will be to oppose the motion of the block. If the block is moving uphill, the frictional force will be downhill. If the block is moving downhill the frictional force will be uphill. If the block is not moving, the frictional force will be only as large as is necessary to balance out the other forces. In this case, the frictional force will be holding the sliding block from sliding up the ramp, so it will be directed downwards.

The frictional force is limited by the magnitude of the perpendicular force between the two surfaces that are sliding, namely:

$$\boxed{F_{\text{friction}} \le \mu F_{\text{normal}}} \qquad \text{(Eq. 3-11)}$$

Most of the time, we are only interested in the extreme case, but we must always be careful to make sure that our frictional forces never cause a stationary object to start moving.

$$\boxed{F_{\text{max-friction}} = \mu F_{\text{normal}}} \qquad \text{(Eq. 3-12)}$$

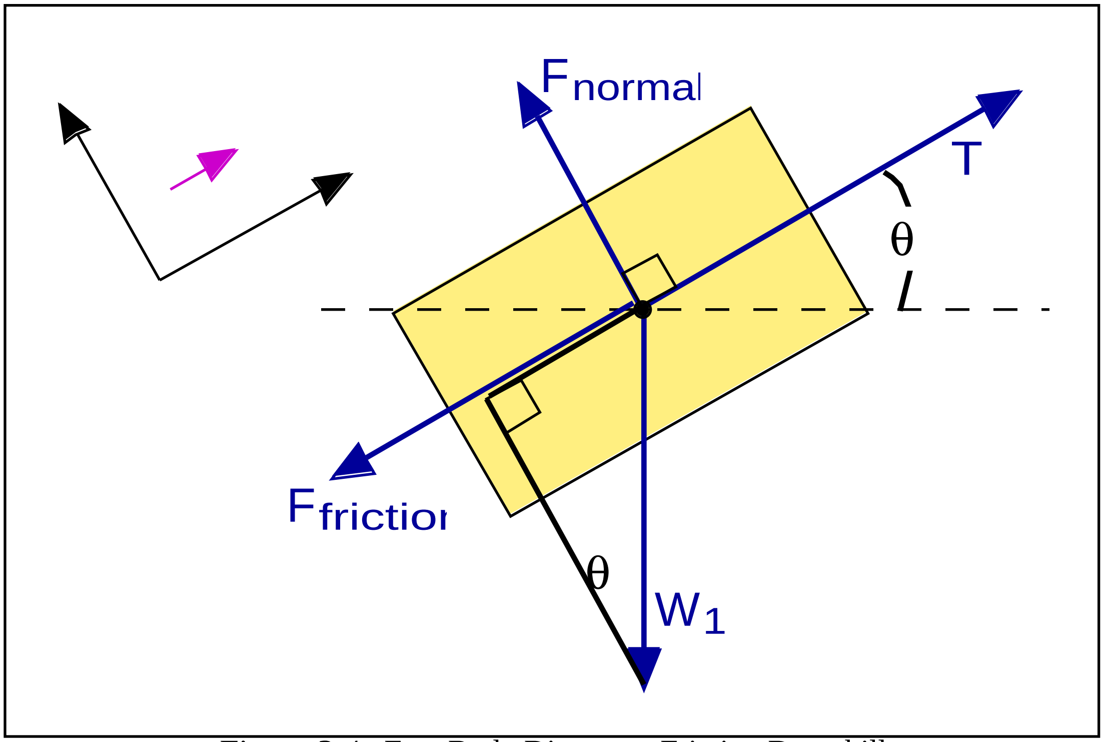

Figure 3-4. Free Body Diagram - Friction Downhill

From the FBD we proceed with Newton's second law:

$$\boxed{\begin{aligned}\sum_i \mathbf{F}_i &= m\mathbf{a}\\ \sum_i F_{x\,i} &= ma_x\\ \sum_i F_{y\,i} &= ma_y\end{aligned}} \qquad \text{(Eq. 3-13)}$$

We know that the acceleration, a, is in the x-direction, with no component in the y-direction, so we get:

$$\boxed{\sum_i F_{x\,i} \quad \Rightarrow \quad T - W_1 \sin\theta - F_{\text{friction}} = m_1 a} \qquad \text{(Eq. 3-14)}$$

$$\boxed{\sum_i F_{x\,i} \quad \Rightarrow \quad F_{\text{normal}} - W_1 \cos\theta = 0} \qquad \text{(Eq. 3-15)}$$

We can use Equation 3-15 and Equation 3-12 to get:

$$\boxed{F_{\text{max-friction}} = \mu F_{\text{normal}} = \mu W_1 \cos\theta} \qquad \text{(Eq. 3-16)}$$

If we put Equation 3-16 into Equation 3-14 we get:

$$\boxed{T - W_1 \sin\theta - \mu W_1 \cos\theta = m_1 a} \qquad \text{(Eq. 3-16)}$$

The FBD and resulting equations for the hanging mass remain as we found in Figure 3-3. Proceeding as before by eliminating T, we eventually get:

$$\boxed{m_2 g - m_2 a - m_1 g \sin\theta - \mu m_1 g \cos\theta = m_1 a} \tag{Eq. 3-17}$$

$$\boxed{a = g\frac{m_2 - m_1(\sin\theta + \mu\cos\theta)}{m_1 + m_2}} \tag{Eq. 3-18}$$

For the equilibrium case, as the question asked, the acceleration is zero which will only occur if:

$$\boxed{m_{2\max} = m_1(\sin\theta + \mu\cos\theta)} \tag{Eq. 3-19}$$

This is the maximum value for the hanging mass that will not cause the block to accelerate up the ramp. If the block is already moving up the ramp, it will continue up the ramp with a constant speed. If the block is moving up the ramp and the hanging mass is less than this value, the block will slow down and stop. If the hanging mass is less than Equation 3- 27 the block will accelerate back down the ramp after stopping, otherwise it will remain stopped.

What about the minimum hanging mass? For this we need to consider the system with the frictional force acting uphill, either because the block is sliding down the hill or because it is stationary and the unbalanced forces would cause the block to slide downhill if there was no friction.

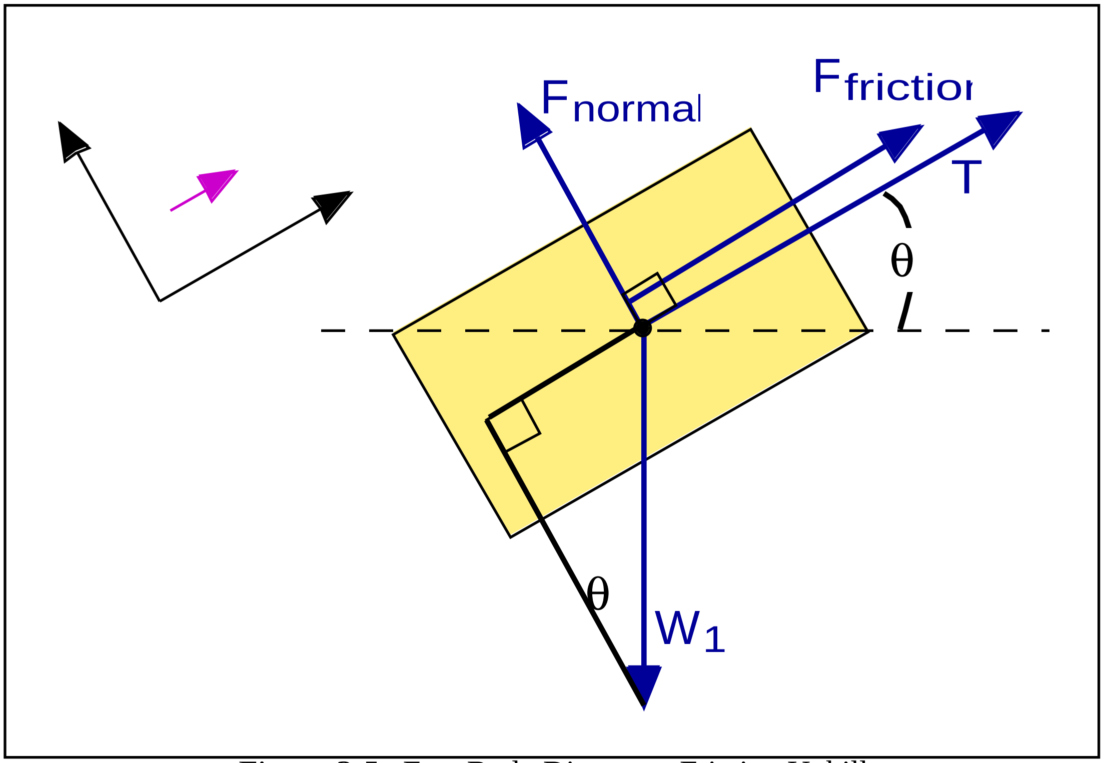

Figure 3-5. Free Body Diagram - Friction Uphill

From the FBD we proceed with Newton's second law:

$$\boxed{\begin{array}{l} \sum_i \mathbf{F}_i = m\mathbf{a} \\ \sum_i F_{x\,i} = ma_x \\ \sum_i F_{y\,i} = ma_y \end{array}} \qquad \text{(Eq. 3-20)}$$

We know that the acceleration, *a*, is in the *x*-direction, with no component in the *y*-direction, so we get:

$$\boxed{\sum_i F_{x\,i} \quad \Rightarrow \quad T - W_1 \sin\theta + F_{\text{friction}} = m_1 a} \qquad \text{(Eq. 3-21)}$$

$$\boxed{\sum_i F_{x\,i} \quad \Rightarrow \quad F_{\text{normal}} - W_1 \cos\theta = 0} \qquad \text{(Eq. 3-22)}$$

We can use Equation 3-15 and Equation 3-22 to get:

$$\boxed{F_{\text{max-friction}} = \mu F_{\text{normal}} = \mu W_1 \cos\theta} \qquad \text{(Eq. 3-23)}$$

If we put Equation 3-23 into Equation 3-21 we get:

$$\boxed{T - W_1 \sin\theta + \mu W_1 \cos\theta = m_1 a} \qquad \text{(Eq. 3-24)}$$

The FBD and resulting equations for the hanging mass remain as we found in Figure 3-3. Proceeding as before by eliminating T, we eventually get:

$$\boxed{m_2 g - m_2 a - m_1 g \sin\theta + \mu m_1 g \cos\theta = m_1 a} \tag{Eq. 3-25}$$

$$\boxed{a = g\frac{m_2 - m_1(\sin\theta - \mu\cos\theta)}{m_1 + m_2}} \tag{Eq. 3-26}$$

For the equilibrium case, as the question asked, the acceleration is zero which will only occur if:

$$\boxed{m_{2\min} = m_1(\sin\theta - \mu\cos\theta)} \tag{Eq. 3-27}$$

This is the minimum value for the hanging mass that will not cause the block to accelerate down the ramp. If the block is already moving down the ramp, it will continue down the ramp with a constant speed. If the block is moving down the ramp and the hanging mass is greater than this value, the block will slow down and stop. If the hanging mass is greater than Equation 3- 19 the block will accelerate back up the ramp after stopping, otherwise it will remain stopped.

Q#4 Circuit

Consider the circuit in Figure 4-1:

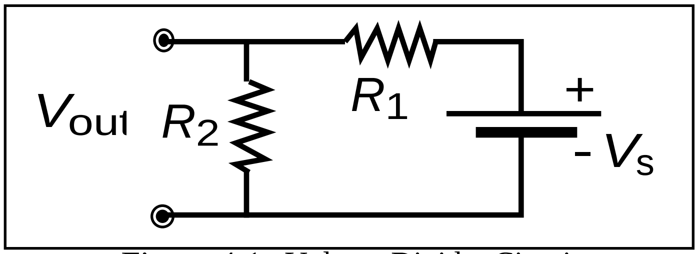

Figure 4-1. Voltage Divider Circuit

A) This circuit is to be used as a voltage divider. The voltage across R_2 should be 1/10 of the source voltage V_s. What value of R_2 (in terms of R_1) will accomplish this? If R_1 is 9000 ohms, what is the value of R_2?

B) If you attach a load with a 1000 ohm impedance to V_{out}, what will the output voltage be?

A#4 Circuit

Consider the circuit in Figure 4-1:

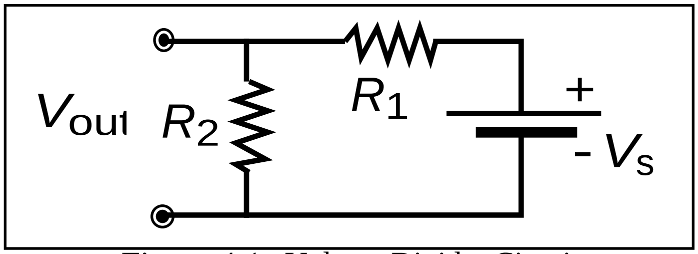

Figure 4-1. Voltage Divider Circuit

A) This circuit is to be used as a voltage divider. The voltage across R_2 should be 1/10 of the source voltage V_S. What value of R_2 (in terms of R_1) will accomplish this? If R_1 is 9000 ohms, what is the value of R_2?

If we perform a counter-clockwise loop around the circuit, the loop rule gives us:

$$\boxed{V_S - IR_1 - IR_2 = 0} \qquad \text{(Eq. 4-1)}$$

$$\boxed{I = \frac{V_S}{R_1 + R_2}} \qquad \text{(Eq. 4-2)}$$

One of our conditions is that:

$$\boxed{V_2 = V_S \frac{1}{10}} \qquad \text{(Eq. 4-3)}$$

This gives us:

$$\boxed{V_2 = IR_2 = V_S \frac{1}{10} \quad \Rightarrow \quad \frac{V_S}{R_1 + R_2} R_2 = V_S \frac{1}{10}} \qquad \text{(Eq. 4-4)}$$

$$\boxed{\begin{aligned} &10\, R_2 = R_1 + R_2 \\ &R_2 = \frac{1}{9} R_2 \end{aligned}} \qquad \text{(Eq. 4-5)}$$

$$\boxed{R_2 = \frac{1}{11} R_1 \quad , \quad R_1 = 9000\ \Omega \quad \Rightarrow \quad R_2 = 1000\ \Omega} \qquad \text{(Eq. 4-6)}$$

B) If you attach a load with a 1000 ohm impedance to V_{out}, what will the output voltage be?

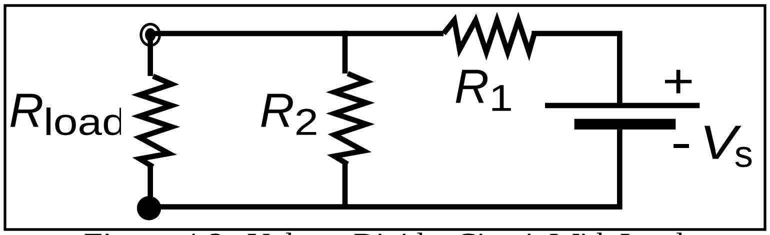

Figure 4-2. Voltage Divider Circuit With Load

We now have a parallel combination between R_2 and R_{load} with a total resistance of:

$$\boxed{\frac{1}{R_{left}} = \frac{1}{R_2} + \frac{1}{R_{load}}} \qquad \text{(Eq. 4-6)}$$

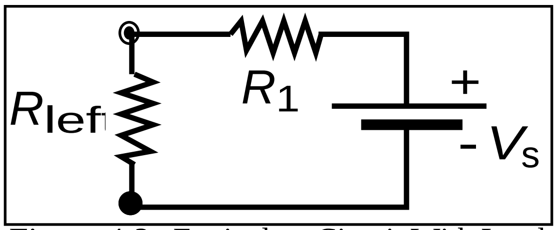

Figure 4-3. Equivalent Circuit With Load

$$\boxed{R_{left} = \frac{R_2 R_{load}}{R_2 + R_{load}}} \qquad \text{(Eq. 4-7)}$$

This gives a current through R_1 or R_{left} of:

$$\boxed{I = \frac{V_S}{R_1 + R_{left}}} \qquad \text{(Eq. 4-8)}$$

The electric potential across R_{left} is equal to the output voltage:

$$\boxed{V_{out} = IR_{left} = V_S \frac{R_{left}}{R_1 + R_{left}}} \qquad \text{(Eq. 4-9)}$$

Putting some numbers into this:

$$\boxed{\begin{aligned} &R_1 = 9000\ \Omega \\ &R_2 = 1000\ \Omega \\ &R_{load} = 1000\ \Omega \\ &R_{left} = 500\ \Omega \\ &V_{out} = V_S \frac{5}{95} = V_S (0.0526315) \end{aligned}} \qquad \text{(Eq. 4-10)}$$

Q#5 Bar In Static Equilibrium

In Figure 5-1 the bar is held at a pivot point on the left and a vertical cable on the right. The bar has uniform density ρ, length *L*, and a cross-sectional area *A*.

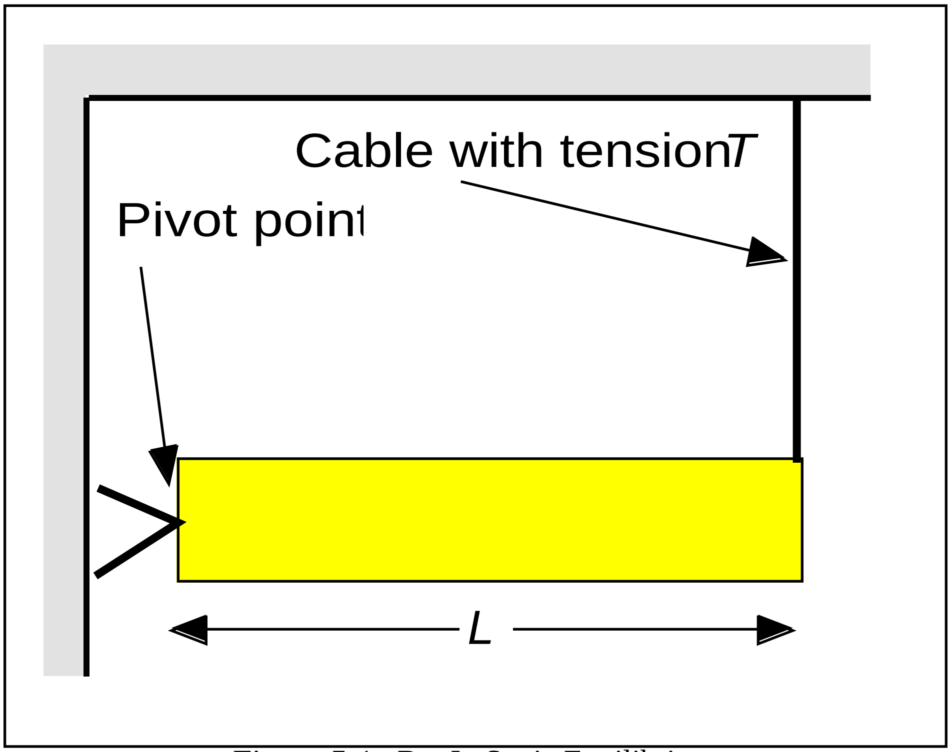

Figure 5-1. Bar In Static Equilibrium

A) What is the tension, *T*, in the cable?

A#5 Bar In Static Equilibrium

In Figure 5-1 the bar is held at a pivot point on the left and a vertical cable on the right. The bar has uniform density ρ, length *L*, and a cross-sectional area *A*.

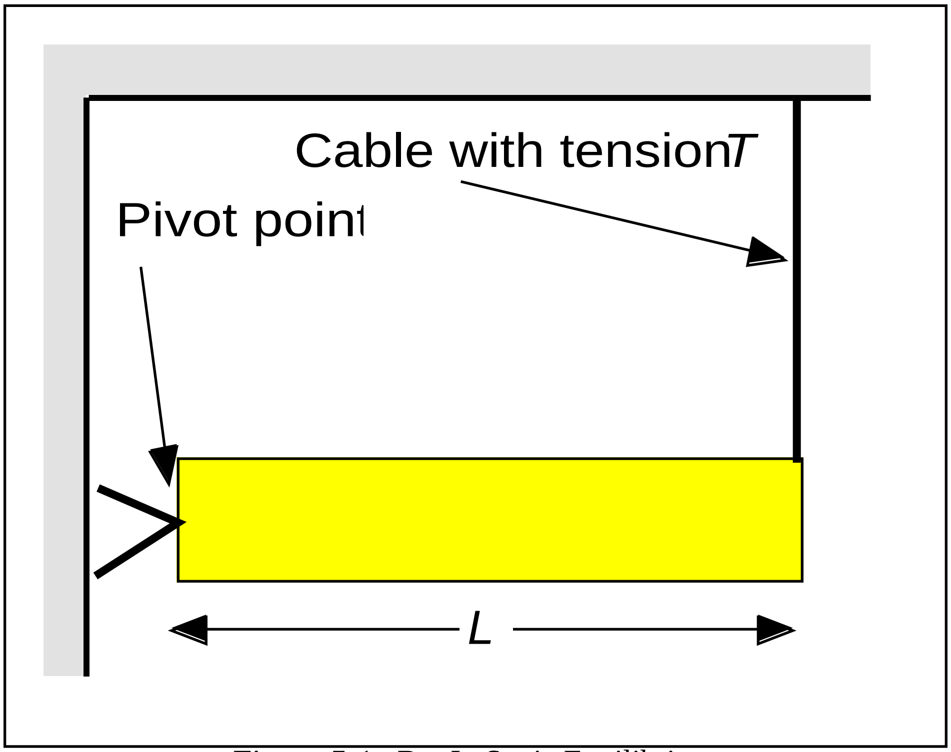

Figure 5-1. Bar In Static Equilibrium

A) What is the tension, *T*, in the cable?

If we assume that the tension *T* is vertical, we can put together a free body diagram for this torque problem. Since we do not know the directions of the forces due to the pivot, we will choose that point about which to calculate our torques. Again we have to define a direction for our coordinate system, in this case positive is clockwise.

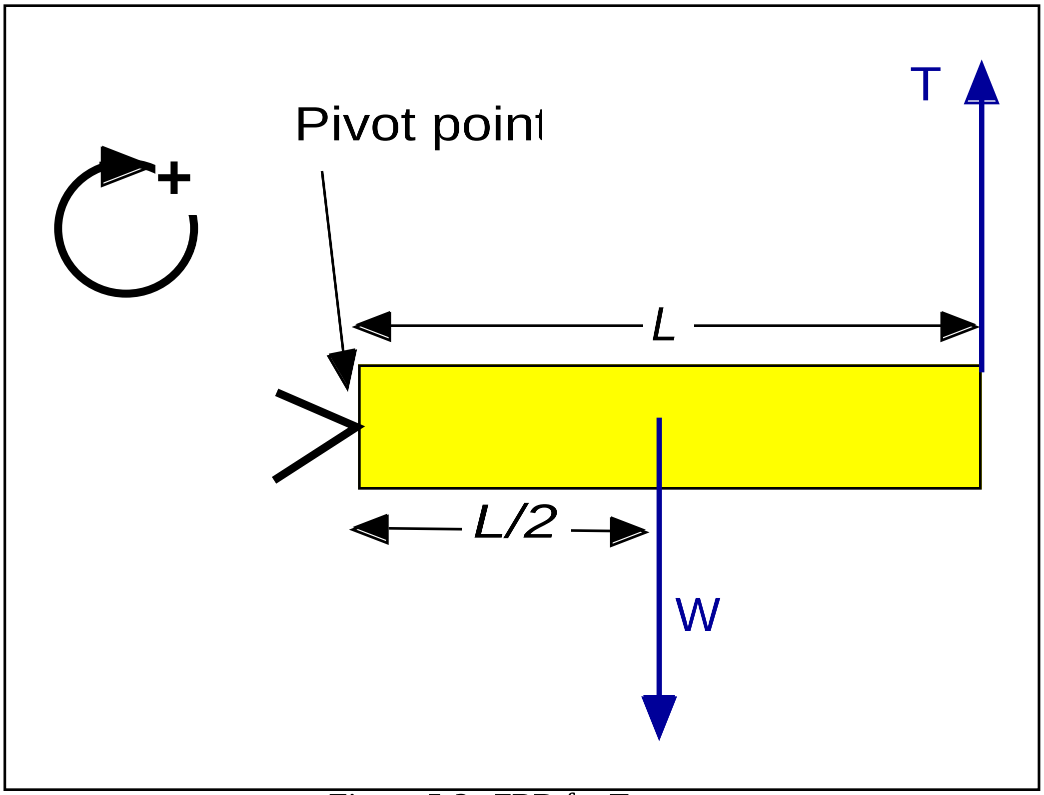

Figure 5-2. FBD for Torques

As with a linear force/acceleration problem we write down the rotational equivalent to Newton's Laws:

$$\boxed{\sum_i \tau_i = I\alpha} \qquad \text{(Eq. 5-1)}$$

Since this is a static problem, the angular acceleration is zero and we have:

$$\boxed{W\frac{L}{2} - TL = 0} \qquad \text{(Eq. 5-2)}$$

The weight of the bar is a function of its volume and density:

$$\boxed{W = \rho V = \rho LA} \qquad \text{(Eq. 5-3)}$$

Giving us the final result of:

$$\boxed{T = \frac{\rho LA}{2}} \qquad \text{(Eq. 5-4)}$$

Q#6 Block Dropped on Spring

A block of mass m is dropped from a height of H_o onto a spring with spring constant k and unstretched length L.

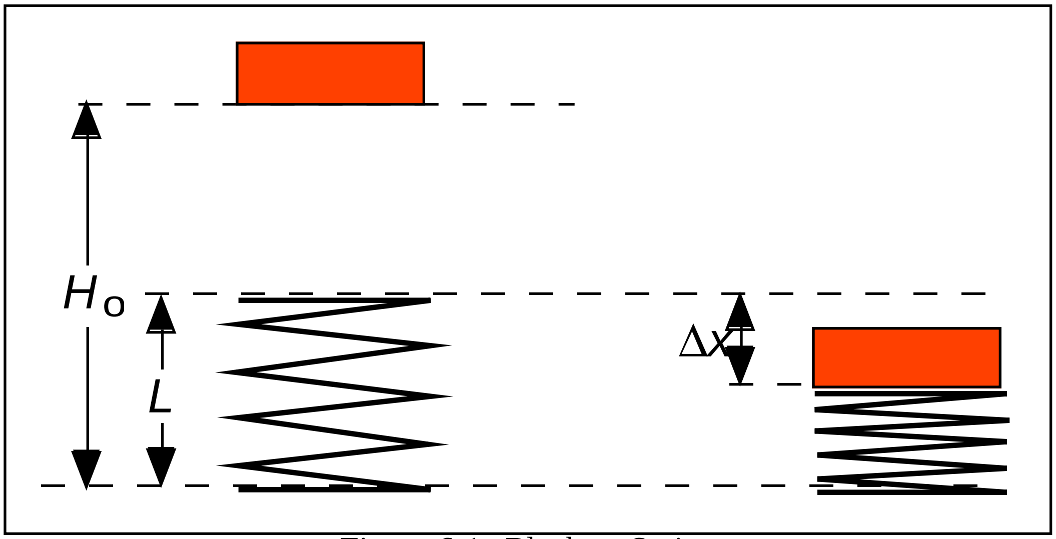

Figure 6-1. Block on Spring

A) What is the maximum compression Δx_A of the spring?

B) If $L = H_o$, what is the maximum compression Δx_B of the spring?

A#6 Block Dropped on Spring

A block of mass m is dropped from a height of H_o onto a spring with spring constant k and unstretched length L.

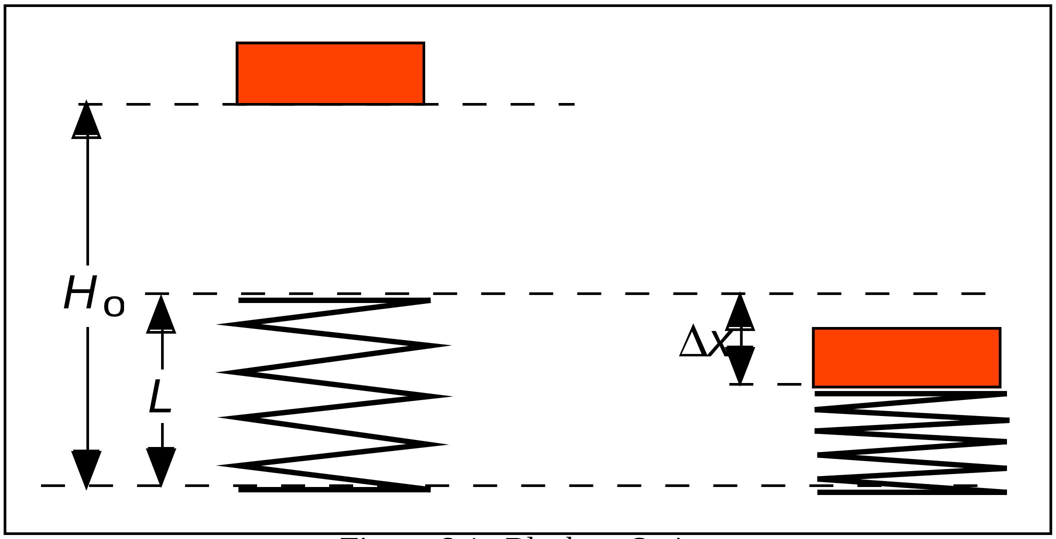

Figure 6-1. Block on Spring

A) What is the maximum compression Δx_A of the spring?

The energy of this system is made up of the kinetic energy of the block, the gravitational potential of the weight, and the potential energy of the compressed spring. No energy was added to or lost from the system in this situation, since the system did not do any work on anything outside the system and there were no internal losses of energy.

$$\boxed{\begin{aligned} &E_i + E_{\text{added}} = E_f + E_{\text{lost}} \\ &PE_{g_i} + PE_{s_i} + KE_i = PE_{g_f} + PE_{s_f} + KE_f \end{aligned}} \qquad \text{(Eq. 6-1)}$$

The block is motionless at the top and the bottom, so the kinetic energy is zero. We have to define a zero value for the gravitational potential energy, which I will take to be zero at the bottom of the spring.

$$\boxed{\begin{aligned} PE_g &= mgh \\ PE_{g_i} &= mgH_o \\ PE_{g_f} &= mg(L - \Delta x) \end{aligned}} \qquad \text{(Eq. 6-2)}$$

The energy stored in the spring is initially zero and when compressed it is:

$$\boxed{\begin{aligned} PE_{s_i} &= 0 \\ PE_{s_f} &= \frac{1}{2}k(\Delta x)^2 \end{aligned}} \qquad \text{(Eq. 6-3)}$$

Putting Equation 6-1, Equation 6-2, and Equation 6-3 together gives us:

$$\boxed{mgH_o = mg(L-\Delta x)+\frac{1}{2}k(\Delta x)^2} \qquad \text{(Eq. 6-4)}$$

$$\boxed{\frac{1}{2}k(\Delta x)^2 - mg\Delta x + mg\left(L-H_o\right)=0} \qquad \text{(Eq. 6-5)}$$

We solve Equation 6-5 via the binomial formula:

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}} \qquad \text{(Eq. 6-6)}$$

$$\boxed{\Delta x=\frac{mg\pm\sqrt{m^2g^2-2kmg\left(L-H_o\right)}}{k}} \qquad \text{(Eq. 6-7)}$$

One of the two solutions given by Equation 6-7 corresponds to the spring being extended rather than compressed, the solution we are looking for is:

$$\boxed{\Delta x=\frac{mg+\sqrt{m^2g^2-2kmg\left(L-H_o\right)}}{k}} \qquad \text{(Eq. 6-8)}$$

B) If $L = H_o$, what is the maximum compression Δx_B of the spring?

Taking Equation 6-8 and setting If $L = H_o$ gives us:

$$\boxed{\Delta x=\frac{2mg}{k}} \qquad \text{(Eq. 6-7)}$$

Note that this is twice the compression that would result from gently placing the block onto the spring - the position where the net force on the block would be zero.

Q#7 Vectors

Consider the following four vectors:

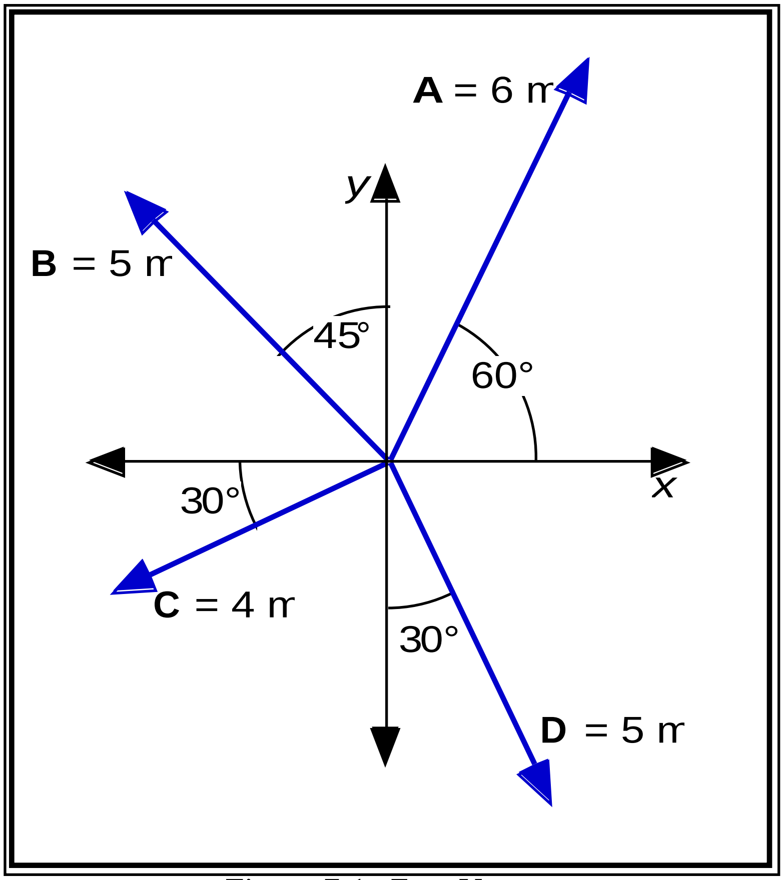

Figure 7-1. Four Vectors

$$\boxed{\begin{array}{ll} \mathbf{A} = 6\text{ m},\ 60^\circ \text{ clockwise from} & x\text{-axis} \\ \mathbf{B} = 5\text{ m},\ 45^\circ \text{ clockwise from} & y\text{-axis} \\ \mathbf{C} = 4\text{ m},\ 30^\circ \text{ clockwise from} & -x\text{-axis} \\ \mathbf{D} = 5\text{ m},\ 60^\circ \text{ clockwise from} & -y\text{-axis} \end{array}} \qquad \text{(Eq. 7-1)}$$

Call their sum the vector $\mathbf{E} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D}$.

A) What are the x and y components of $\mathbf{E}$?

B) What angle does $\mathbf{E}$ make with the x-axis?

A#7 Vectors

Consider the following four vectors:

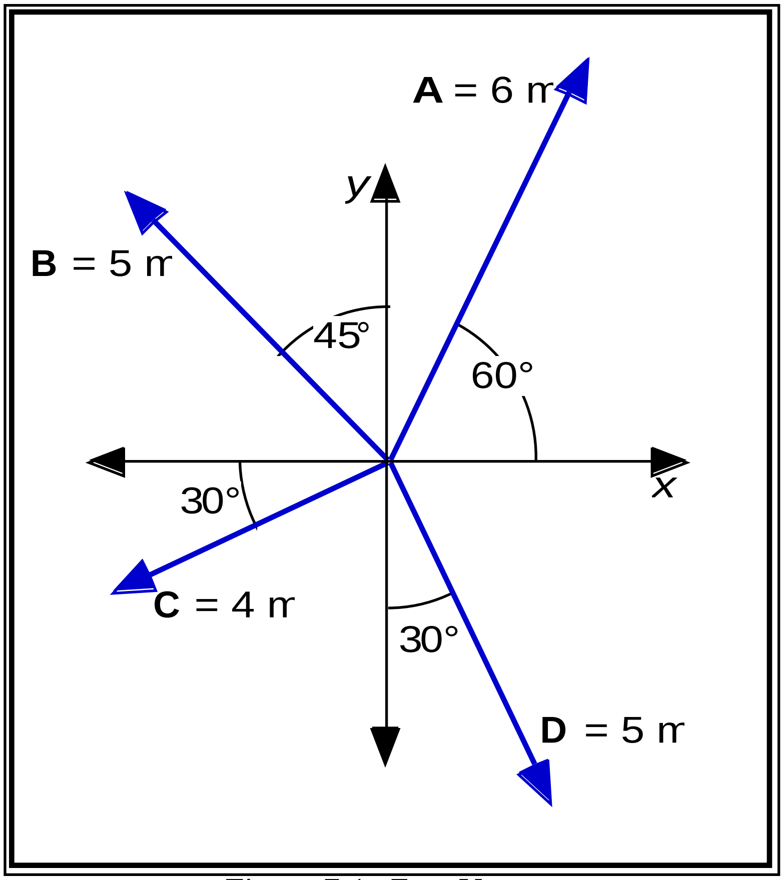

Figure 7-1. Four Vectors

$$\boxed{\begin{array}{l} \mathbf{A} = 6\text{ m},\ 60^\circ \text{ clockwise from } x\text{-axis} \\ \mathbf{B} = 5\text{ m},\ 45^\circ \text{ clockwise from } y\text{-axis} \\ \mathbf{C} = 4\text{ m},\ 30^\circ \text{ clockwise from } -x\text{-axis} \\ \mathbf{D} = 5\text{ m},\ 60^\circ \text{ clockwise from } -y\text{-axis} \end{array}} \qquad \text{(Eq. 7-1)}$$

Call their sum the vector $\mathbf{E} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D}$.

A) What are the x and y components of **E**?

Trigonometry gives us the components of the vectors. $A_x = A\cos\theta$, $A_y = A\sin\theta$ for example.

$$\boxed{\begin{aligned}\mathbf{A} &= (3.000 \quad 5.196)\,\mathrm{m}\\ \mathbf{B} &= (-3.54 \quad 3.54)\,\mathrm{m}\\ \mathbf{C} &= (-3.46 \quad -2.00)\,\mathrm{m}\\ \mathbf{D} &= (2.50 \quad -4.33)\,\mathrm{m}\end{aligned}} \qquad \text{(Eq. 7-2)}$$

The sum of the vectors in Equation 7-2 is:

$$\boxed{\mathbf{E} = (-1.4996 \quad 2.40155)\,\mathrm{m}} \qquad \text{(Eq. 7-3)}$$

B) What angle does **E** make with the x-axis?

$$\boxed{\tan\theta = \frac{E_y}{E_x}} \qquad \text{(Eq. 7-4)}$$

$$\boxed{\begin{gathered}\tan\theta = \frac{2.40155}{-1.4996} = -1.60146 \quad \Rightarrow \quad \theta = -58.018^\circ\\ \theta = 58.018^\circ \text{ counter - clockwise from } -x\text{ - axis}\end{gathered}} \qquad \text{(Eq. 7-5)}$$

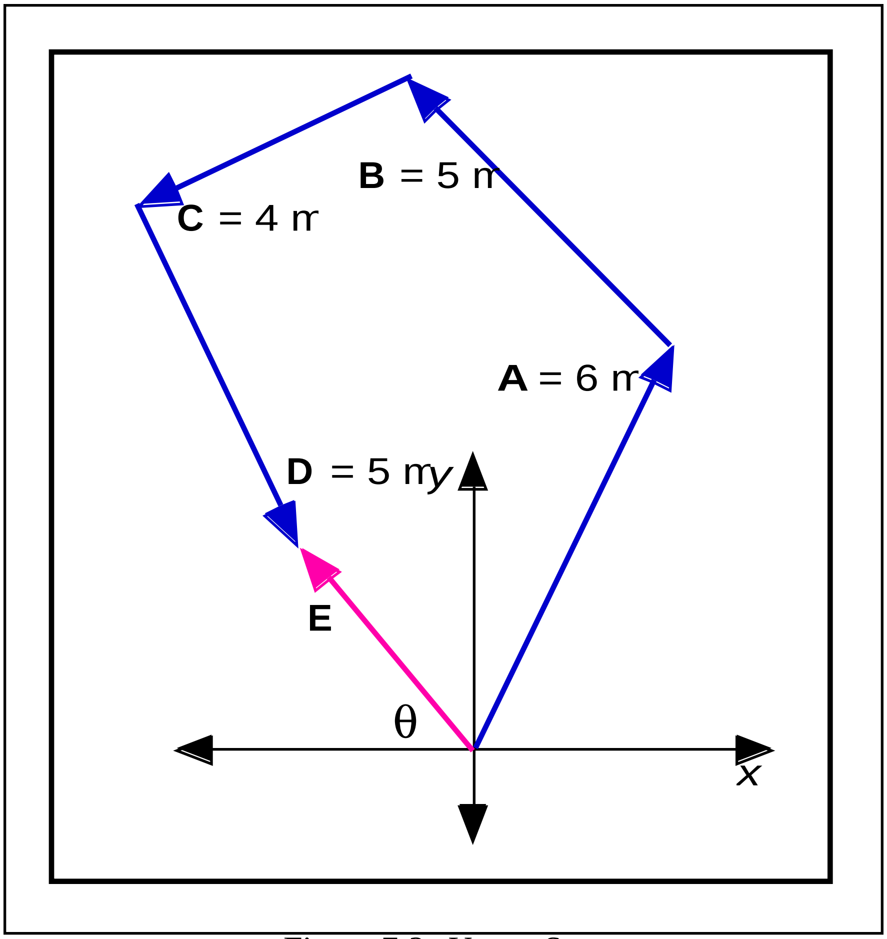

Figure 7-2. Vector Sum

Q#8 Standing Wave

Consider a standing wave on a 1 meter long wire rigidly at both ends. The tension in the wire is 1000 N and its mass per unit length is 0.1 kg/m.

A) What is the wave number and angular frequency of the second harmonic vibration of the wire?

B) Write the displacement of the wire from it's equilibrium position as a function of the distance along the wire and time.

C) How many nodes exist for this vibration?

A#8 Standing Wave

Consider a standing wave on a 1 meter long wire rigidly at both ends. The tension in the wire is 1000 N and its mass per unit length is 0.1 kg/m.

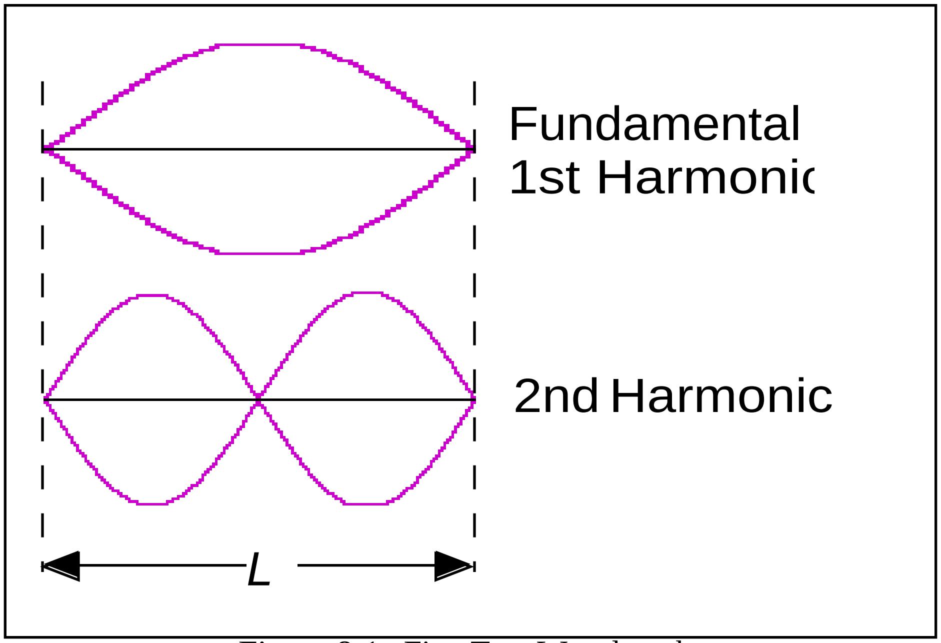

Figure 8-1. First Two Wavelengths

A) What is the wave number and angular frequency of the second harmonic vibration of the wire?

The speed of a wave traveling on a string is:

$$\boxed{v = f\lambda = \frac{\omega}{k} = \sqrt{\frac{F_{\text{Tension}}}{\mu}}} \quad \text{(Eq. 8-1)}$$

The angular frequency ω, is related to the period T and the frequency f by:

$$\boxed{f = \frac{1}{T} = \frac{\omega}{2\pi}} \quad \text{(Eq. 8-2)}$$

The wavelength λ is related to the wave number k by the relation:

$$\boxed{k = \frac{2\pi}{\lambda}} \quad \text{(Eq. 8-3)}$$

If we put these together we get:

$$\boxed{\omega = 2\pi k v = 2\pi \frac{2\pi}{\lambda}\sqrt{\frac{F_{\text{Tension}}}{\mu}} = \frac{4\pi^2}{\lambda}\sqrt{\frac{F_{\text{Tension}}}{\mu}}} \quad \text{(Eq. 8-4)}$$

The wavelength of the 2nd harmonic, as can be seen in Figure 8-1 is the length of the string. So since we were given:

$$\boxed{\begin{aligned} &F_{\text{Tension}} = 1000\ \text{N} \\ &\mu = 0.1\frac{\text{kg}}{\text{m}} \\ &L = 1\ \text{m} \\ &\lambda = L = 1\ \text{m} \end{aligned}} \qquad \text{(Eq. 8-5)}$$

From the above we have:

$$\boxed{k = \frac{2\pi}{\lambda} = 6.2832\frac{1}{\text{m}}} \qquad \text{(Eq. 8-6)}$$

$$\boxed{\omega = \frac{4\pi^2}{\lambda}\sqrt{\frac{F_{\text{Tension}}}{\mu}} = 3947.84\frac{\text{rad}}{\text{sec}} = 3947.84\ \text{Hz}} \qquad \text{(Eq. 8-7)}$$

B) Write the displacement of the wire from it's equilibrium position as a function of the distance along the wire and time.

A standing wave consists of two waves, one going in each direction. The two waves would have displacement expressions such as:

$$\boxed{\begin{aligned} y_{+x\ \text{direction}} &= A_+ \sin(kx - \omega t) \\ y_{-x\ \text{direction}} &= A_- \sin(kx + \omega t) \end{aligned}} \qquad \text{(Eq. 8-8)}$$

We can add these two together and combine them using the trigonometric addition rules to arrive at:

$$\boxed{y_{\text{standing}} = B\sin(kx)\cos(\omega t)} \qquad \text{(Eq. 8-9)}$$

In Equation 8-9, B is the amplitude of the wave, the height of the highest points, halfway between adjacent nodes.

C) How many nodes exist for this vibration?

As in Figure 8-1, the first harmonic has two nodes, the second harmonic has three, with each higher harmonic having one more node.

Q#9 Electrostatics

What is the energy in electron volts (eV) required to move a 10^{-16} C charge from a distance of 10^{-3} cm to a distance of 10^{-4} cm from the surface of a charged sphere. The charge on the sphere is 10^{-12} C/cm^2 and the radius of the sphere is 10^{-2} cm?

A#9 Electrostatics

What is the energy in electron volts (eV) required to move a 10^{-16} C charge from a distance of 10^{-3} cm to a distance of 10^{-4} cm from the surface of a charged sphere. The charge on the sphere is 10^{-12} C/cm^2 and the radius of the sphere is 10^{-2} cm?

First off we need to put everything into units that we can work with. Note that the initial and final positions of the moving charge are given from the surface of the sphere, not from the center, thus we have to convert them.

$$\boxed{\begin{aligned} R_{\text{sphere}} &= 10^{-2}\,\text{cm} = 10^{-4}\,\text{m} \\ r_i &= 10^{-2}\,\text{cm} + 10^{-3}\,\text{cm} = 1.1\cdot 10^{-4}\,\text{m} \\ r_f &= 10^{-2}\,\text{cm} + 10^{-4}\,\text{cm} = 1.01\cdot 10^{-4}\,\text{m} \end{aligned}} \quad \text{(Eq. 9-1)}$$

$$\boxed{q = 10^{-16}\,\text{C} = 625\text{ e}} \quad \text{(Eq. 9-2)}$$

We need to find the charge on the sphere by multiplying the surface charge density by the surface area.

$$\boxed{\begin{aligned} \sigma_{\text{sphere}} &= 10^{-12}\,\frac{\text{C}}{\text{cm}^2} = 10^{-8}\,\frac{\text{C}}{\text{m}^2} \\ A_{\text{sphere}} &= 4\pi R_{\text{sphere}}^2 \\ Q_{\text{sphere}} &= \sigma_{\text{sphere}} A_{\text{sphere}} = 1.2566\cdot 10^{-15}\,\text{C} = 7854\text{ e} \end{aligned}} \quad \text{(Eq. 9-3)}$$

We want to find the electric potential difference between the initial and final points. We know it will be positive since the sphere has a net positive charge.

$$\boxed{k_e = 9\times 10^9\,\frac{\text{Vm}}{\text{C}}} \quad \text{(Eq. 9-4)}$$

$$\boxed{\Delta V = k_e\frac{Q}{r_f} - k_e\frac{Q}{r_i} = k_e Q\left(\frac{1}{r_f} - \frac{1}{r_i}\right)} \quad \text{(Eq. 9-5)}$$

$$\boxed{\begin{aligned} \Delta V &= 9.162\cdot 10^{-3}\,\frac{\text{J}}{\text{C}} \\ &= 9.162\cdot 10^{-3}\text{ V} \end{aligned}} \quad \text{(Eq. 9-6)}$$

The energy required to move the charge will be equal to the charge's change in electric potential energy, which we can find by multiplying the electric potential difference between the two points by the value of the charge being moved.

$$\boxed{U = q\Delta V} \quad \text{(Eq. 9-7)}$$

$$\boxed{\begin{aligned} U &= 5.726 \text{ eV} \\ &= 9.161 \cdot 10^{-19} \text{ J} \end{aligned}} \qquad \text{(Eq. 9-8)}$$

Q#10 Gas and Pressure

Two cylindrical containers of lengths $L_1 > L_2$ are separated by a valve. The containers have the same radius r and both have a temperature of 25°C.

A) If container 1 is at pressure P_1 and container P_2 is evacuated what will be the pressure in both containers after the valve is opened?

B) If the temperature is raised 300°C what is the final pressure in the system?

C) What is the force on the walls of the containers (assume there is a vacuum outside of the cylinders and neglect any effects of the valve)?

A#10 Gas and Pressure

Two cylindrical containers of lengths $L_1 > L_2$ are separated by a valve. The containers have the same radius r and both have a temperature of 25°C.

A) If container 1 is at pressure P_1 and container P_2 is evacuated what will be the pressure in both containers after the valve is opened?

The volume of each cylinder is given by:

$$\boxed{\begin{aligned} V_1 &= \pi r^2 L_1 \\ V_2 &= \pi r^2 L_2 \end{aligned}} \qquad \text{(Eq. 10-1)}$$

The pressure, volume and temperature are all related to the number of particles of gas in the system and the gas constant R.

$$\boxed{\begin{aligned} &PV = nRT \\ &\frac{PV}{T} = nR = \text{constant} \end{aligned}} \qquad \text{(Eq. 10-2)}$$

The temperature must be measured in Kelvin, from absolute zero. Since there is no work done on expansion into the larger volume, and there is no change in the thermal energy of the system, the temperature remains constant.

$$\boxed{\begin{array}{ll} T_i = (25 + 273.15)\text{ K} & T_f = T_i \\ \quad = 298.15\text{ K} & \\ V_i = V_1 & V_f = V_1 + V_2 \\ P_i = P_1 & P_f = ? \end{array}} \qquad \text{(Eq. 10-3)}$$

The relationship between the pressures and volumes is thus:

$$\boxed{P_i V_i = P_f V_f} \qquad \text{(Eq. 10-4)}$$

$$\boxed{P_f = \frac{P_i V_i}{V_f} = \frac{P_1 \pi r^2 L_1}{\pi r^2 (L_1 + L_2)} = \frac{P_1 L_1}{(L_1 + L_2)}} \qquad \text{(Eq. 10-5)}$$

B) If the temperature is raised 300°C what is the final pressure in the system?

$$\begin{array}{|ll|}\hline T_i = (25 + 273.15)\,\mathrm{K} & T_f = (300 + 273.15)\,\mathrm{K} \\ \quad = 298.15\,\mathrm{K} & \quad = 573.15\,\mathrm{K} \\ V_i = V_1 & V_f = V_1 + V_2 \\ P_i = P_1 & P_f = ? \\ \hline \end{array} \qquad \text{(Eq. 10-6)}$$

The relationship between the pressures, temperatures, and volumes is thus:

$$\boxed{\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f}} \qquad \text{(Eq. 10-7)}$$

$$\boxed{P_f = \frac{P_i V_i T_f}{V_f T_i} = \frac{P_1 \pi r^2 L_1 T_f}{\pi r^2 (L_1 + L_2) T_i} = \frac{P_1 L_1 T_f}{(L_1 + L_2) T_i}} \qquad \text{(Eq. 10-8)}$$

$$\begin{array}{|l|}\hline P_f = \dfrac{P_1 L_1}{(L_1 + L_2)} \dfrac{573.15}{298.15} \\ \quad = \dfrac{P_1 L_1}{(L_1 + L_2)} 1.92235 \\ \hline \end{array} \qquad \text{(Eq. 10-9)}$$

C) What is the force on the walls of the containers (assume there is a vacuum outside of the cylinders and neglect any effects of the valve)?

The force on the walls is the product of the pressure and the area of the wall.

For the initial case, the pressure is P_1 in the first container, and zero in the second container, so the forces on the walls of the second container are zero. The forces on *each* end piece and of the cylinder sides are all outward of magnitude:

$$\begin{array}{|ll|}\hline F_{1\text{ each end}} = \pi r^2 P_1 & F_{1\text{ cylinder wall}} = 2\pi r L_1 P_1 \\ F_{2\text{ each end}} = \pi r^2 P_2 = 0 & F_{2\text{ cylinder wall}} = 2\pi r L_2 P_2 = 0 \\ \hline \end{array} \qquad \text{(Eq. 10-10)}$$

For the case in part (A), we now have only a single longer cylinder. The forces on *each* end piece and of the cylinder sides are all outward of magnitude:

$$\boxed{\begin{aligned} F_{\text{each end}} &= \pi r^2 P_f = \frac{\pi r^2 P_1 L_1}{(L_1 + L_2)} \\ F_{\text{cylinder wall}} &= 2\pi r(L_1 + L_2)P_f = 2\pi r P_1 L_1 \end{aligned}} \qquad \text{(Eq. 10-11)}$$

For the case in part (D), we still have only a single longer cylinder, only the pressure is now higher by a factor of 1.92235. The forces on *each* end piece and of the cylinder sides are all outward of magnitude:

$$\boxed{\begin{aligned} F_{\text{each end}} &= \pi r^2 P_f = 1.92235\frac{\pi r^2 P_1 L_1}{(L_1 + L_2)} \\ F_{\text{cylinder wall}} &= 2\pi r(L_1 + L_2)P_f = 1.92235 \cdot 2\pi r P_1 L_1 \end{aligned}} \qquad \text{(Eq. 10-12)}$$